Synthesis

Reading Reflection

Discuss in groups

- How would the different synthesis approaches described in the reading affect the user interaction model?
- How would the approaches described in the reading apply or not apply to the various synthesis project ideas you brainstormed during our first synthesis week?

Reading Key Takeaways

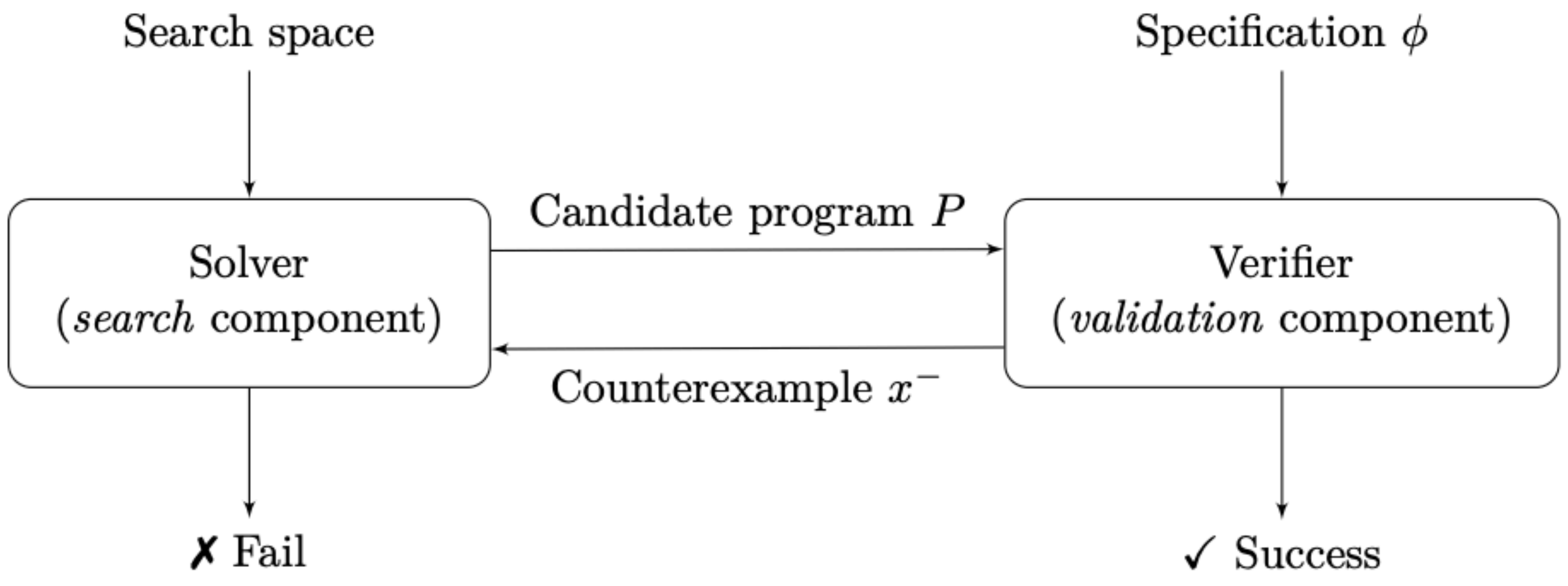


Figure 3.2: Counterexample-guided inductive synthesis.

- CEGIS!

- Distinguishing inputs—2 programs match our spec. How will we find the one we want? Ask the user what we should do on this next input, for which the programs produce different outputs.
- Syntactic bias—as we've already discussed, language shapes the search space
- SyGuS—SyGuS solvers can be a really useful starting point for a new synthesis project! See Fig 3.10 for how nice the programs are.

SyGuS string example

```
(set-logic SLIA)

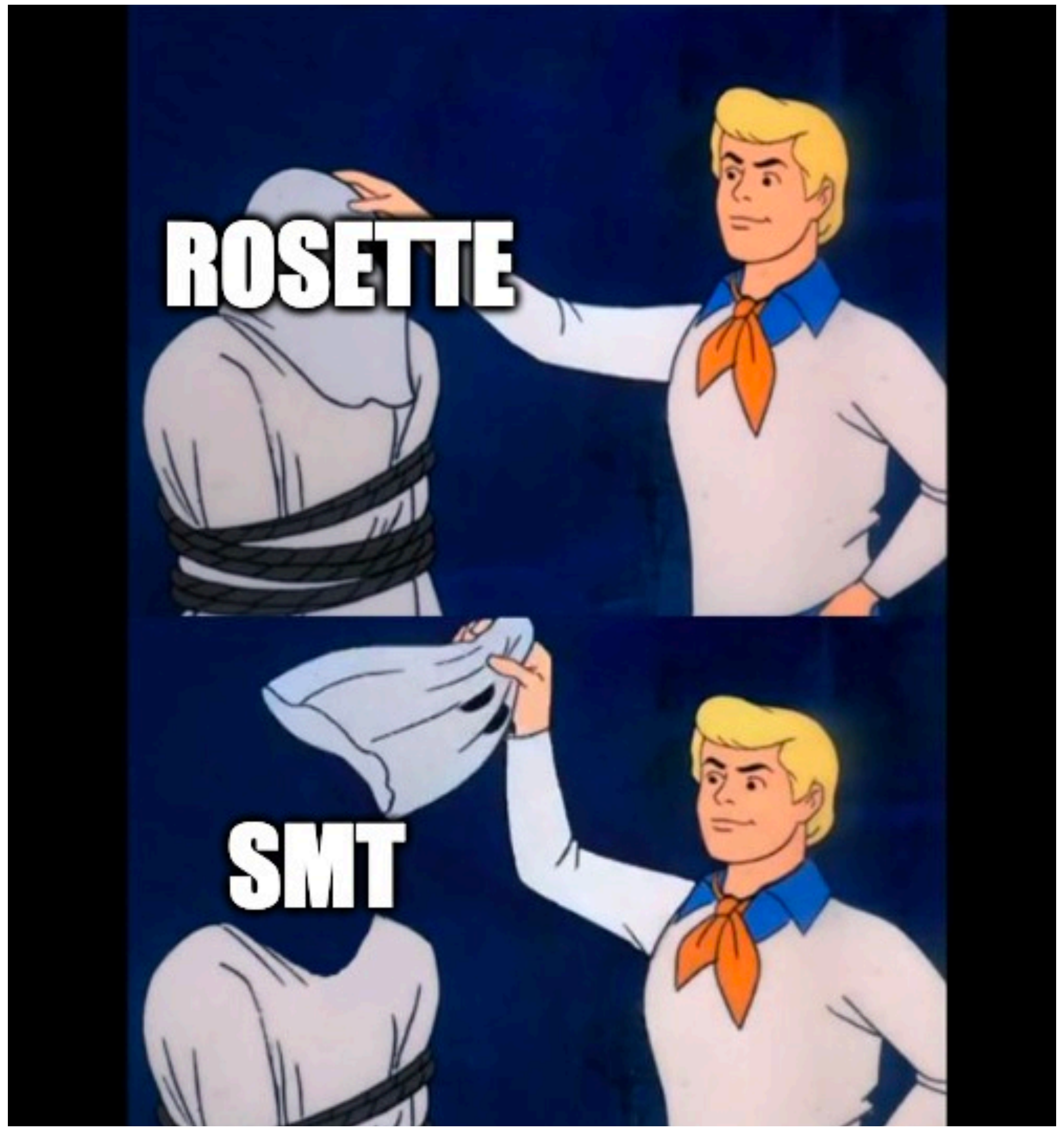
(synth-fun f ((name String)) String
    ((Start String (ntString))
    (ntString String (name " " "." "Dr." (str.++ ntString ntString)
        (str.replace ntString ntString ntString) (str.at ntString ntInt) (int.to.str ntInt)
        (str.substr ntString ntInt ntInt)))
    (ntInt Int (0 1 2 (+ ntInt ntInt) (- ntInt ntInt) (str.len ntString)
        (str.to.int ntString) (str.indexof ntString ntString ntInt)))
    (ntBool Bool (true false (str.prefixof ntString ntString)
        (str.suffixof ntString ntString) (str.contains ntString ntString)))))

(declare-var name String)
(constraint (= (f "Nancy FreeHafer") "Dr. Nancy"))
(constraint (= (f "Andrew Cencici") "Dr. Andrew"))
(constraint (= (f "Jan Kotas") "Dr. Jan"))
(constraint (= (f "Mariya Sergienko") "Dr. Mariya"))

(check-synth)
```

Reflections on Rosette

- Concise program -> quite complex and sophisticated synthesizers
- Opacity
- Control


ROSETTE
SMT

Today's topic: SMT

You

Spec

Answers!

The Rosette Language

Logical Constraints

Answers!

SMT Solvers

OK, what's SMT?

Satisfiability Modulo Theories

ok, and what's
satisfiability??

Let’s back up

- **SAT: Boolean satisfiability problem**; also sometimes called SATISFIABILITY
  - Given a Boolean formula, is there an interpretation of the formula that satisfies it? **Can we replace the variables of the Boolean formula with the values TRUE or FALSE such that the formula evaluates to TRUE?**
    - If yes, the formula is satisfiable
    - If no assignment out of all possible assignments makes the formula TRUE, it’s unsatisfiable
- Examples:
  - $p \wedge q$ is satisfiable; (p=TRUE, q=TRUE)
  - $p \wedge \neg p$ is unsatisfiable
- SAT is NP-complete
  - …but that hasn’t stopped folks from building some seriously efficient SAT solvers and using them to solve real problems

Next few slides shamelessly lifted from…

Computer-Aided Reasoning for Software

CSE507

SAT Solving Basics

Emina Torlak
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See https://courses.cs.washington.edu/courses/cse507/19au/calendar.html for more

Syntax of propositional logic

$$(\neg \boldsymbol{p} \wedge \top) \vee (\boldsymbol{q} \rightarrow \bot)$$

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Atom — **truth symbols**: $\top$ (“true”), $\bot$ (“false”)
propositional variables: $p, q, r, \ldots$

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Literal	an atom α or its negation $\neg\alpha$

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Atom	**truth symbols**: $\top$ ("true"), $\bot$ ("false") **propositional variables**: $p, q, r, \ldots$
Literal	an atom α or its negation $\neg\alpha$
Formula	an atom or the application of a **logical connective** to formulas F_1, F_2:

$\neg F_1$	"not"	(negation)
$F_1 \wedge F_2$	"and"	(conjunction)
$F_1 \vee F_2$	"or"	(disjunction)
$F_1 \rightarrow F_2$	"implies"	(implication)
$F_1 \leftrightarrow F_2$	"if and only if"	(iff)

Semantics of propositional logic: interpretations

An **interpretation** I for a propositional formula F maps every variable in F to a truth value:

$$I : \{\, p \mapsto \text{true}, q \mapsto \text{false}, \ldots\}$$

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I is a **satisfying interpretation** of F, written as $I \models F$, if F evaluates to true under I.

I is a **falsifying interpretation** of F, written as $I \not\models F$, if F evaluates to false under I.

A satisfying interpretation is also called a **model**.

Satisfiability & validity of propositional formulas

F is **satisfiable** iff $I \models F$ for some I.

F is **valid** iff $I \models F$ for all I.

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Duality of satisfiability and validity:

F is valid iff $\neg F$ is unsatisfiable.

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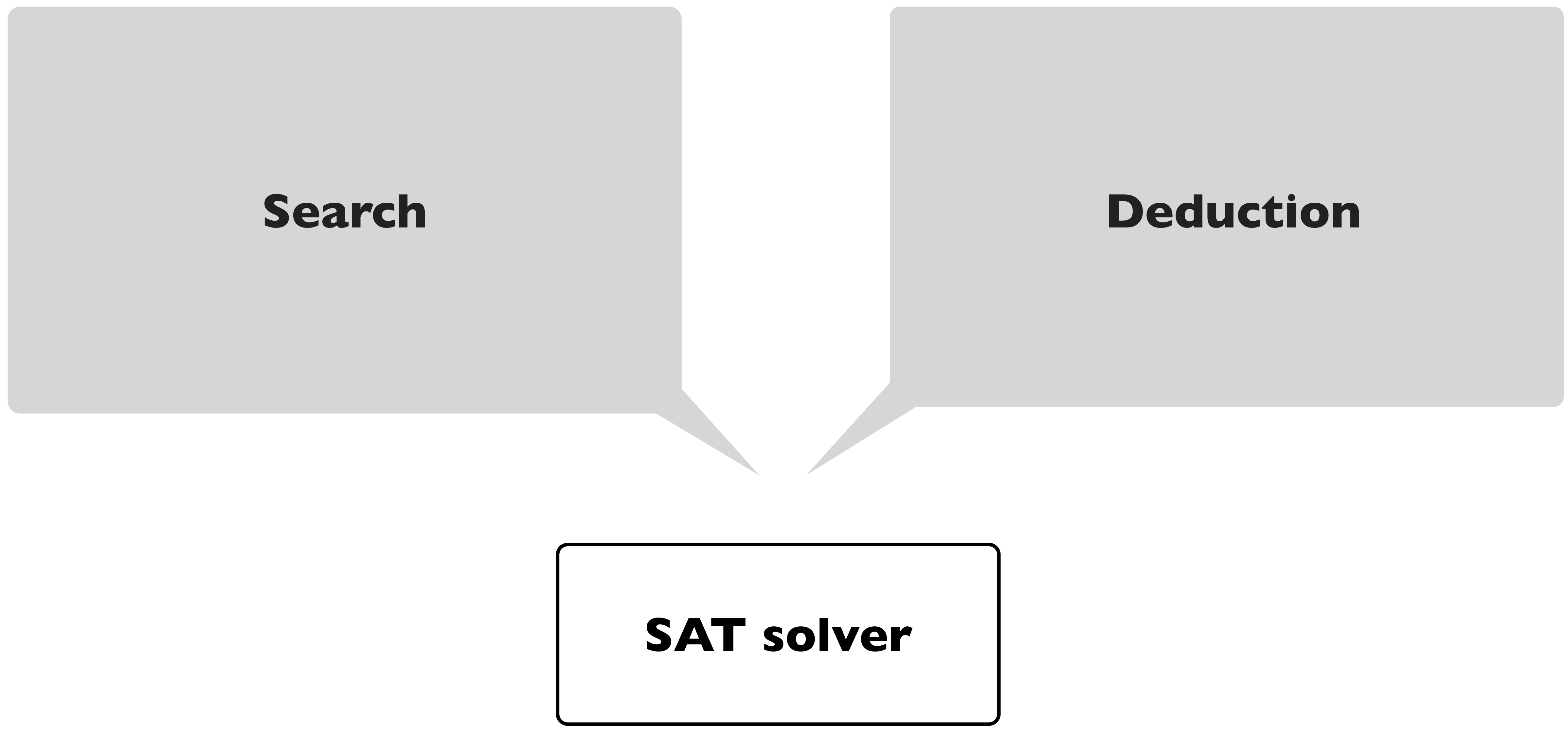
If we have a procedure for checking satisfiability, we can also check validity of propositional formulas, and vice versa.

Techniques for deciding satisfiability & validity



Techniques for deciding satisfiability & validity

Search

Enumerate all interpretations (i.e., build a truth table), and check that they satisfy the formula.

Deduction

Assume the formula is invalid, apply proof rules, and check for contradiction in every branch of the proof tree.

SAT solver

Proof by search: enumerating interpretations

F: $(p \wedge q) \rightarrow (p \vee \neg q)$

p	q	$p \wedge q$	$\neg q$	$p \vee \neg q$	F
0	0	0	1	1	1
0	1	0	0	0	1
1	0	0	1	1	1
1	1	1	0	1	1

Valid.

5 minute break

Now that we know about SAT…

- Ok so…what’s SMT?
- Satisfiability (SAT) **Modulo Theories**

Next few slides shamelessly lifted from…

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Satisfiability Modulo Theories

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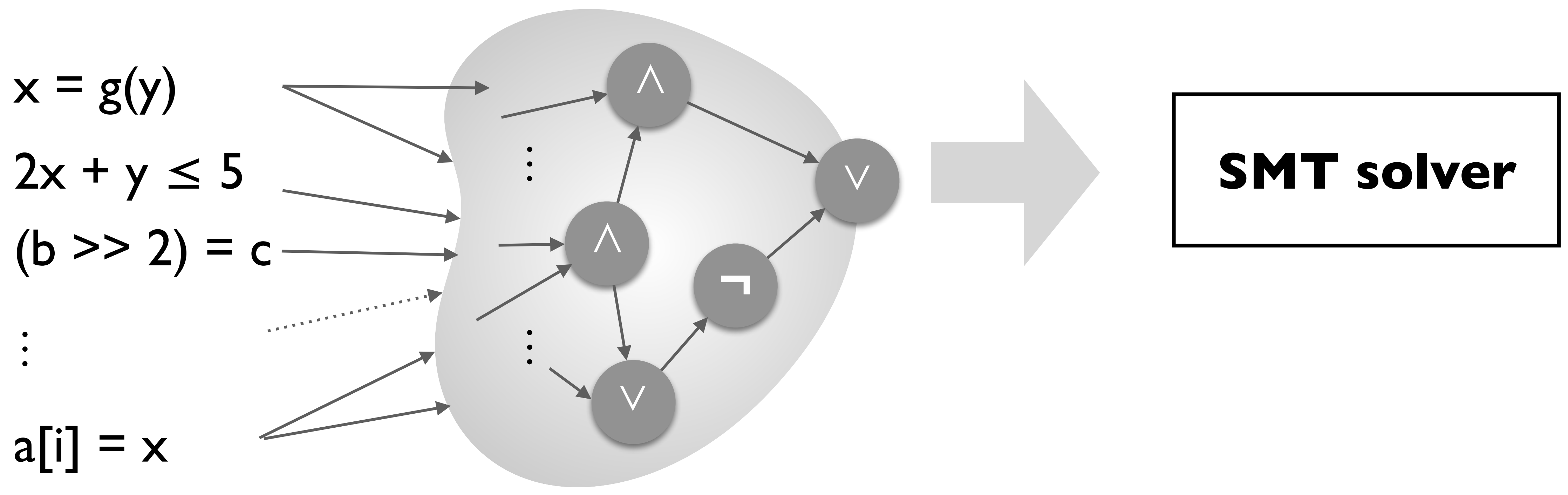
Satisfiability Modulo Theories (SMT)



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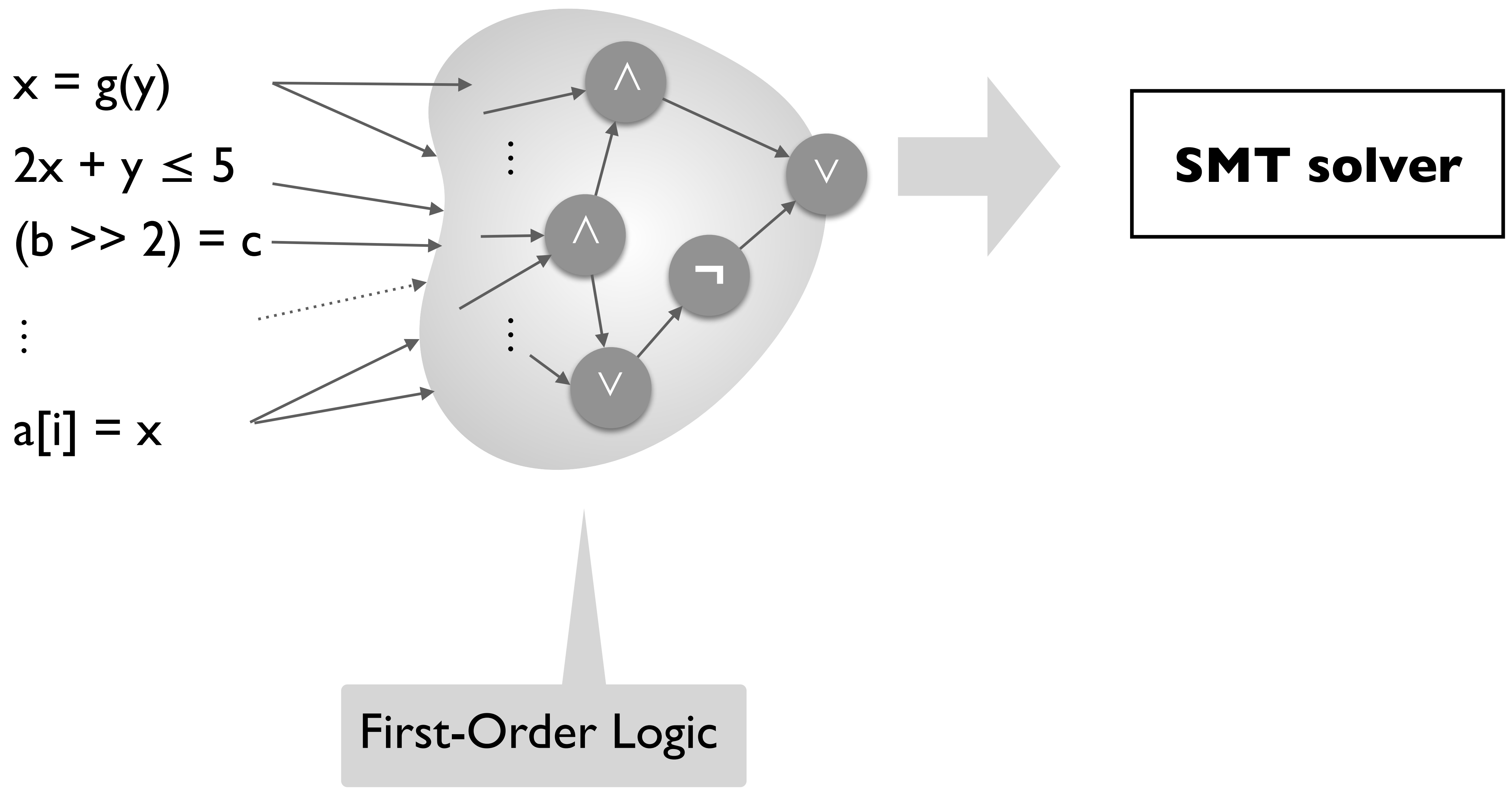

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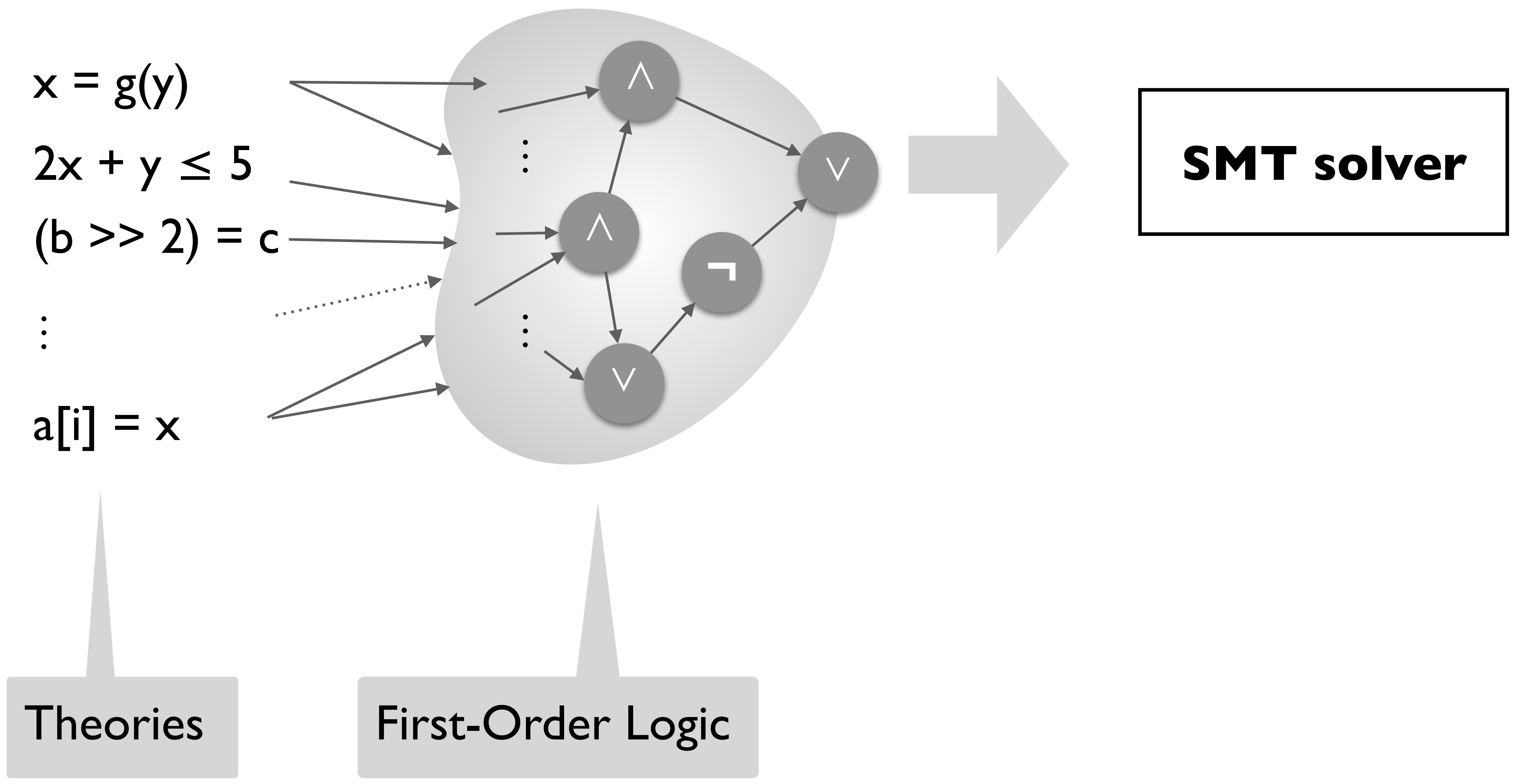

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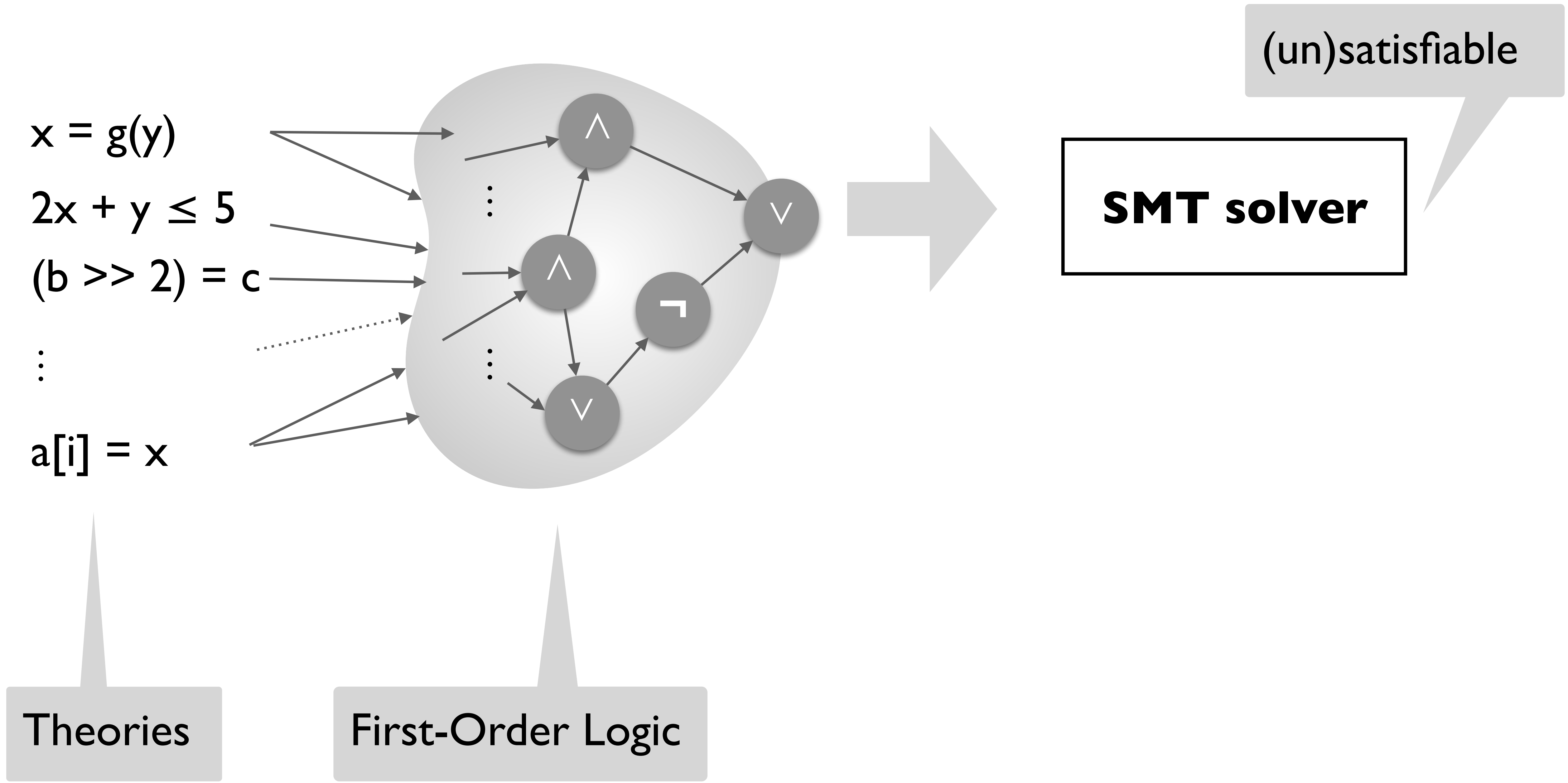

Syntax of First-Order Logic (FOL)

Logical symbols

- Connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- Parentheses: ()
- ✗ Quantifiers: $\forall, \exists$

Non-logical symbols

- Constants: x, y, z
- N-ary functions: f, g
- N-ary predicates: p, q
- ✗ Variables: u, v, w

We will only consider the **quantifier-free** fragment of FOL.

In particular, we will consider quantifier-free **ground** formulas.

Semantics of FOL: example

Universe

- A non-empty set of values
- Finite or (un)countably infinite

Interpretation

- Maps a constant symbol c to an element of U: $I[c] \in U$
- Maps an n-ary function symbol f to a function $f_I : U^n \rightarrow U$
- Maps an n-ary predicate symbol p to an n-ary relation $p_I \subseteq U^n$

U = {☀, ☁}

I[x] = ☀

I[y] = ☁

I[f] = {☀ ↦ ☁, ☁ ↦ ☀}

I[p] = {⟨☀,☀⟩, ⟨☀,☁⟩}

$\langle U, I \rangle \vDash p(f(y), f(f(x)))$?

You decide!
Take 1 min.

Satisfiability and validity of FOL

F is **satisfiable** iff $M \models F$ for some structure $M = \langle U, I \rangle$.

F is **valid** iff $M \models F$ for all structures $M = \langle U, I \rangle$.

Duality of satisfiability and validity:

F is valid iff $\neg F$ is unsatisfiable.

Common theories

Equality (and uninterpreted functions)

- x = g(y)

Fixed-width bitvectors

- (b >> 1) = c

Linear arithmetic (over R and Z)

- $2x + y \leq 5$

Arrays

- a[i] = x

Theory of equality with uninterpreted functions

Signature: {=, x, y, z, …, f, g, …, p, q, …}

- The binary predicate = is *interpreted*.
- All constant, function, and predicate symbols are *uninterpreted.*

Axioms

- $\forall x.\ x = x$
- $\forall x, y.\ x = y \rightarrow y = x$
- $\forall x, y, z.\ x = y \wedge y = z \rightarrow x = z$
- $\forall x_1, \ldots, x_n, y_1, \ldots, y_n.\ (x_1 = y_1 \wedge \ldots \wedge x_n = y_n) \rightarrow (f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n))$
- $\forall x_1, \ldots, x_n, y_1, \ldots, y_n.\ (x_1 = y_1 \wedge \ldots \wedge x_n = y_n) \rightarrow (p(x_1, \ldots, x_n) \leftrightarrow p(y_1, \ldots, y_n))$

Deciding $T_=$

- Conjunctions of literals modulo $T_=$ is decidable in polynomial time.

$T_=$ example: checking program equivalence

```
int abs(int y) {
  return y<0 ? -y : y;
}

int sq(int y) {
  return y*y;
}

int sqabs(int y) {
  return abs(y)*abs(y);
}
```

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Are **sq** and **sqabs** equivalent on all 128-bit integers?

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Yes, but the solver takes a while to return an answer because reasoning about multiplication is expensive.

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Are **sq** and **sqabs** equivalent on all 128-bit integers?

Yes, but the solver takes a while to return an answer because reasoning about multiplication is expensive.

What happens if we replace the multiplication with an uninterpreted function?

Theory of fixed-width bitvectors

Signature

- Fixed-width words modeling machine ints, longs, …
- Arithmetic operations: bvadd, bvsub, bvmul, …
- Bitwise operations: bvand, bvor, bvnot, …
- Comparison predicates: bvlt, bvgt, …
- Equality: =
- Expanded with all constant symbols: x, y, z, …

Deciding T_{BV}

- NP-complete.

Theories of linear integer and real arithmetic

Signature

- Integers (or reals)
- Arithmetic operations: multiplication by an integer (or real) number, +, -.
- Predicates: =, $\leq$.
- Expanded with all constant symbols: x, y, z, …

Deciding T_{LIA} and T_{LRA}

- NP-complete for linear integer arithmetic (LIA).
- Polynomial time for linear real arithmetic (LRA).
- Polynomial time for difference logic (conjunctions of the form $x - y \leq c$, where c is an integer or real number).

LIA example: compiler optimization

```
for (i=1; i<=10; i++) {
  a[j+i] = a[j];
}
```

```
int v = a[j];
for (i=1; i<=10; i++) {
  a[j+i] = v;
}
```

A LIA formula that is unsatisfiable iff this transformation is valid:

LIA example: compiler optimization

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for (i=1; i<=10; i++) {
  a[j+i] = a[j];
}
```

```
int v = a[j];
for (i=1; i<=10; i++) {
  a[j+i] = v;
}
```

A LIA formula that is unsatisfiable iff this transformation is valid:

$(i \geq 1) \wedge (i \leq 10) \wedge$

$(j + i = j)$

Theory of arrays

Signature

- Array operations: read, write
- Equality: =
- Expanded with all constant symbols: x, y, z, …

Axioms

- $\forall a, i, v.\ read(write(a, i, v), i) = v$
- $\forall a, i, j, v.\ \neg(i = j) \rightarrow (read(write(a, i, v), j) = read(a, j))$
- $\forall a, b.\ (\forall i.\ read(a, i) = read(b, i)) \rightarrow a = b$

Deciding T_A

- Satisfiability problem: NP-complete.
- Used in many software verification tools to model memory.

Basically…

- SAT lets us say simple things
- SMT lets us say…other simple things. But more complicated than SAT!
    - And it's enough that we can get to some interesting tasks

SMT activity